

IEEE International Conference on Rebooting Computing (ICRC 2022)

San Francisco, California, USA, December 8-9, 2022

Reprinted for International Symposium on Roadmapping Devices & Systems (ISRDS '23)

May 3-4, 2023



Sandia  
National  
Laboratories

# Ballistic Asynchronous Reversible Computing in Superconducting Circuits



Thursday, May 4<sup>th</sup>, 2023

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Approved for public release, SAND2022-16982 C



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# Contributors to our Reversible Computing research program

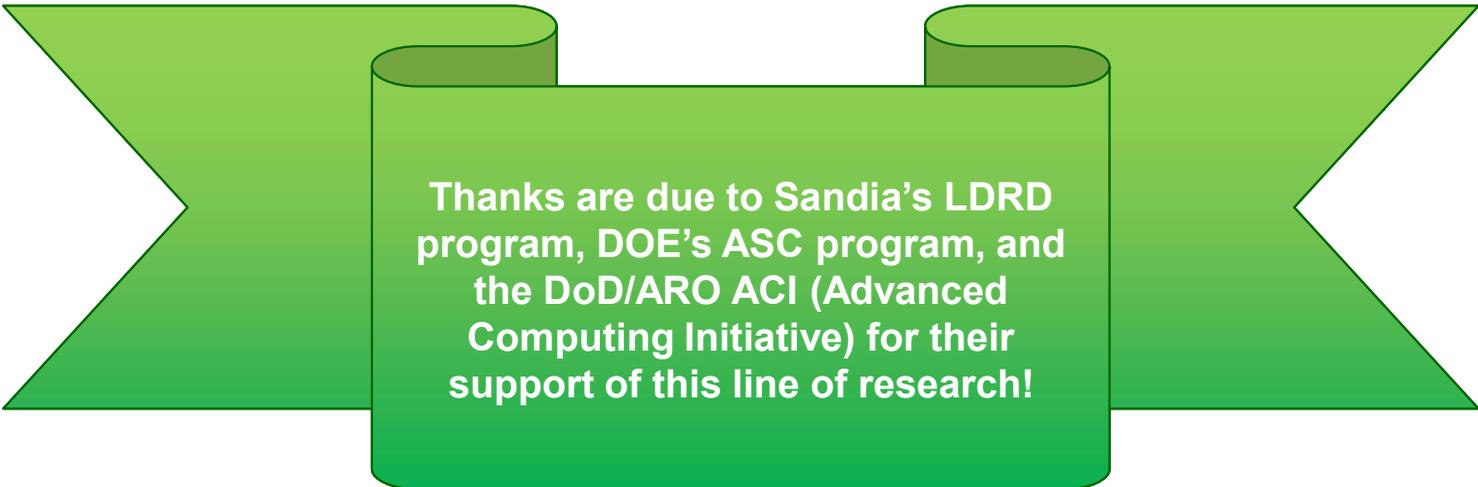


## ■ Full group of recent staff at Sandia:

- Michael Frank (Cognitive & Emerging Computing)
- Robert Brocato (RF MicroSystems) – now retired
- David Henry (MESA Hetero-Integration)
- Rupert Lewis (Quantum Phenomena)
  - Terence “Terry” Michael Bretz-Sullivan
- Nancy Missert (Nanoscale Sciences) – now retired
  - Matt Wolak (now at Northrop-Grumman)
- Brian Tierney (Rad Hard CMOS Technology)

## ■ Thanks are also due to the following colleagues & external research collaborators:

- Karpur Shukla (CMU → Flame U. → Brown U.)
  - Currently in Prof. Jimmy Xu’s Lab for Emerging Techs.
- Hannah Earley (Cambridge U. → startup)
- Erik DeBenedictis (Sandia → Zettaflops, LLC)
- Joseph Friedman (UT Dallas)
  - with A. Edwards, X. Hu, B.W. Walker, F. Garcia-Sanchez, P. Zhou, J.A.C. Incorviaz, A. Paler
- Kevin Osborn (LPS/JQI)
  - Liuqi Yu, Ryan Clarke, Han Cai
- Steve Kaplan (independent contractor)
- Rudro Biswas (Purdue)
  - Dewan Woods & Rishabh Khare
- Tom Conte (Georgia Tech/CRNCH)
  - Anirudh Jain, Gibran Essa
- David Guéry-Odelin (Toulouse U.)
- FAMU-FSU College of Engineering:
  - Sastry Pamidi (ECE Chair) & Jerris Hooker (Instructor)
  - 2019-20 students:
    - Frank Allen, Oscar L. Corces, James Hardy, Fadi Matloob
  - 2020-21 students:
    - Marshal Nachreiner, Samuel Perlman, Donovan Sharp, Jesus Sosa

A large green ribbon graphic with a white border, containing text. The ribbon is shaped like a wide 'X' with a central rectangular section.

Thanks are due to Sandia’s LDRD program, DOE’s ASC program, and the DoD/ARO ACI (Advanced Computing Initiative) for their support of this line of research!

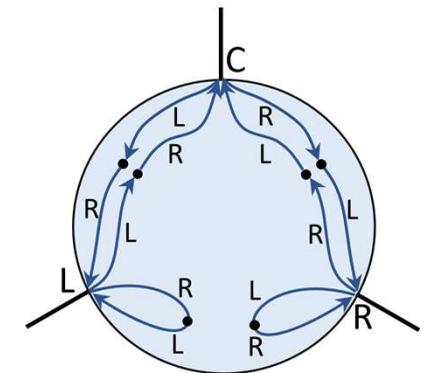
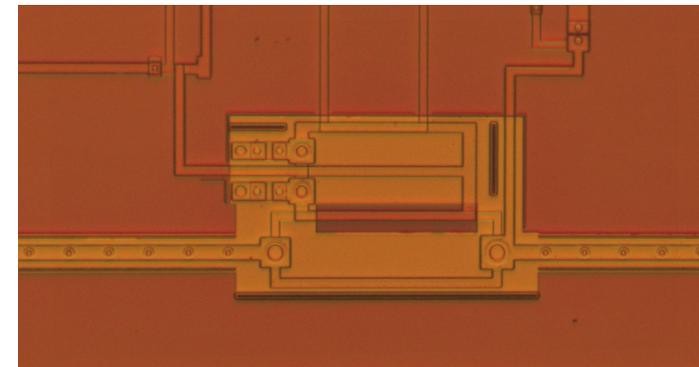
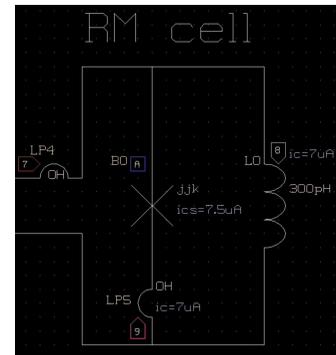
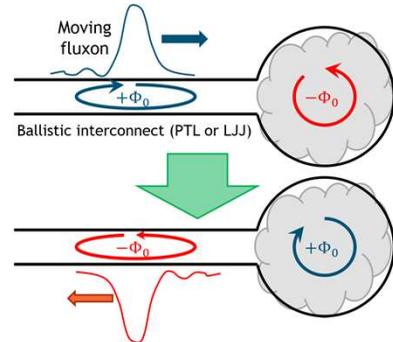
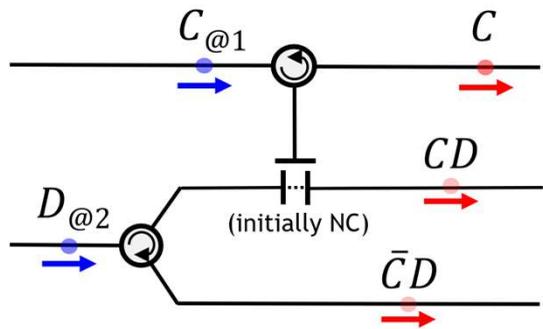
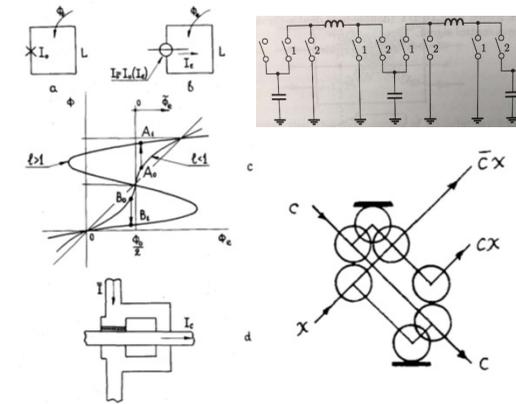
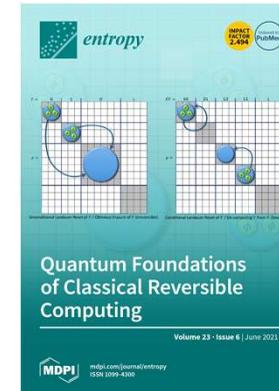
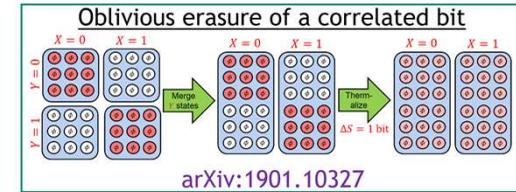
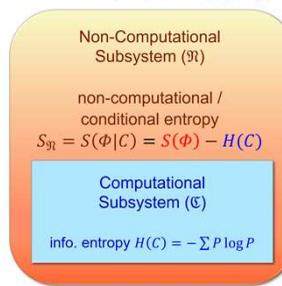


# Talk Abstract/Outline

## Ballistic Asynchronous Reversible Computing in Superconducting Circuits

- Background: *Why Reversible Computing?*
  - Relevant classic results in the thermodynamics of computing
    - Recently generalized to quantum case
  - Two major types of approaches to reversible computing in superconducting circuits:
    - *Adiabatic approaches* – Well-developed today.
      - Likharev's *parametric qnantron* (1977); more recent QFP tech (YNU & collabs.) w. substantial demo chips.
    - *Ballistic approaches* – Much less mature to date.
      - Fredkin & Toffoli's early concepts (1978-'81); much more recent work at U. Maryland, Sandia, UC Davis
- **Review:** The relatively new *asynchronous* ballistic approach to RC in SCE.
  - Addresses concerns w instability of the synchronous ballistic approach
  - Potential advantages of asynchronous ballistic RC (vs. adiabatic approaches)
  - Implementation w. superconducting circuits (BARCS effort).
- **Focus of this Talk:**
  - Presenting our recent work on enumerating/classifying possible BARCS functions w.  $\leq 3$  ports and  $\leq 2$  states.

Computing System ( $\mathcal{C}$ ),  
total entropy  $S(\Phi) = -\sum p \log p$



# Ballistic Reversible Computing

Can we envision reversible computing as a deterministic elastic interaction process?

Historical origin of this concept:

- Fredkin & Toffoli's *Billiard Ball Model* of computation ("Conservative Logic," IJTP 1982).
  - Based on elastic collisions between moving objects.
  - Spawned a subfield of "collision-based computing."
    - Using localized pulses/solitons in various media.

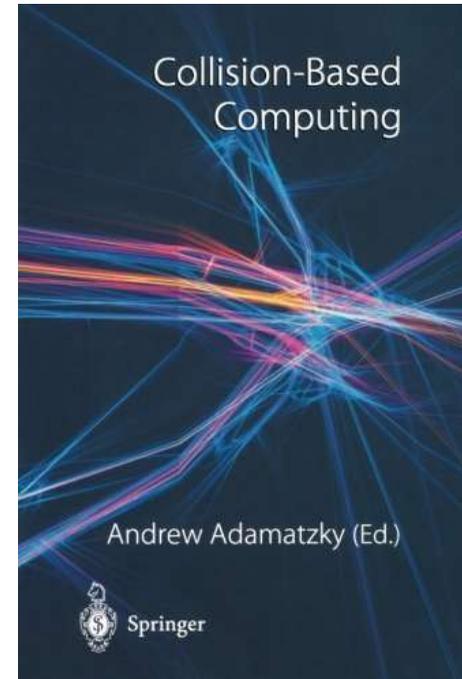
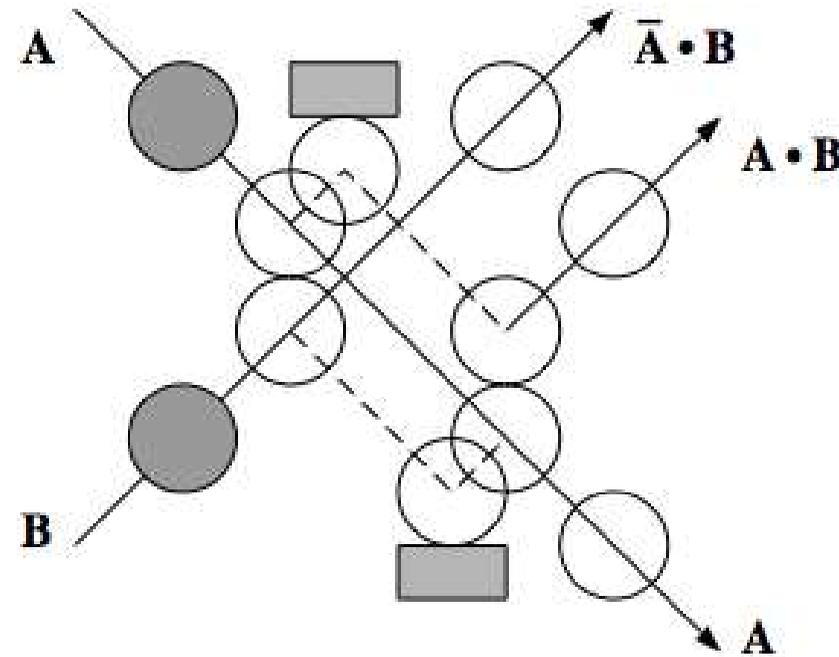
No power-clock driving signals needed!

- Devices operate when data signals arrive.
- The operation energy is carried by the signal itself.
  - Most of the signal energy is preserved in outgoing signals.

However, all (or almost all) of the existing design concepts for ballistic computing invoke implicitly *synchronized* arrivals of ballistically-propagating signals...

- Making this work in reality presents some serious difficulties, however:
  - Unrealistic in practice to assume precise alignment of signal arrival times.
    - Thermal fluctuations & quantum uncertainty, at minimum, are always present.
  - Any relative timing uncertainty leads to chaotic dynamics when signals interact.
    - Exponentially-increasing uncertainties in the dynamical trajectory.
  - Deliberate *resynchronization* of signals whose timing relationship is uncertain incurs an inevitable energy cost.

Can we come up with a new ballistic model that avoids these problems?



# Ballistic Asynchronous Reversible Computing (BARC)



**Problem:** Conservative (dissipationless) dynamical systems generally tend to exhibit chaotic behavior...

- This results from direct nonlinear *interactions* between multiple continuous dynamical degrees of freedom (DOFs), which amplify uncertainties, exponentially compounding them over time...
- *E.g.*, positions/velocities of ballistically-propagating “balls”
  - Or more generally, any localized, cohesive, momentum-bearing entity: Particles, pulses, quasiparticles, solitons...

**Core insight:** In principle, we can greatly reduce or eliminate this tendency towards dynamical chaos...

- We can do this simply by *avoiding* any direct interaction between continuous DOFs of different ballistically-propagating entities

Require localized pulses to arrive *asynchronously*—and furthermore, at clearly distinct, *non-overlapping* times

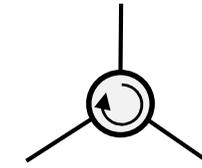
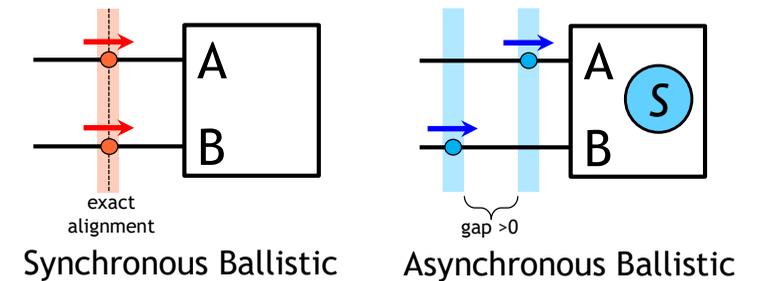
- Device’s dynamical trajectory then becomes *independent* of the precise (absolute *and* relative) pulse arrival times
  - As a result, timing uncertainty per logic stage can now accumulate only *linearly*, not exponentially!
    - Only relatively occasional re-synchronization will be needed
- For devices to still be capable of doing logic, they must now maintain an internal discrete (digitally-precise) state variable—a stable (or at least metastable) stationary state, *e.g.*, a ground state of a well

No power-clock signals, unlike in adiabatic designs!

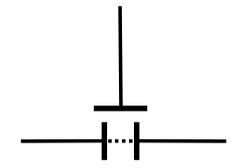
- Devices simply operate whenever data pulses arrive
- The operation energy is carried by the pulse itself
  - Most of the energy is preserved in outgoing pulses
    - Signal restoration can be carried out incrementally

**Goal of current effort at Sandia:** Demonstrate BARC principles in an implementation based on fluxon dynamics in Superconducting Electronics (SCE)

(BARCS  effort)

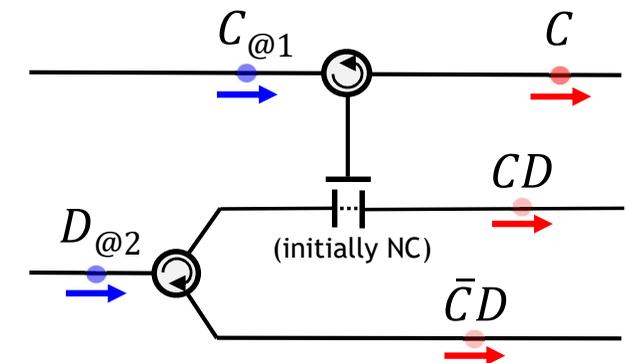


Rotary  
(Circulator)



Toggled  
Barrier

Example BARC device functions



Example logic construction

# Simplest Fluxon-Based (bipolarized) BARC Function

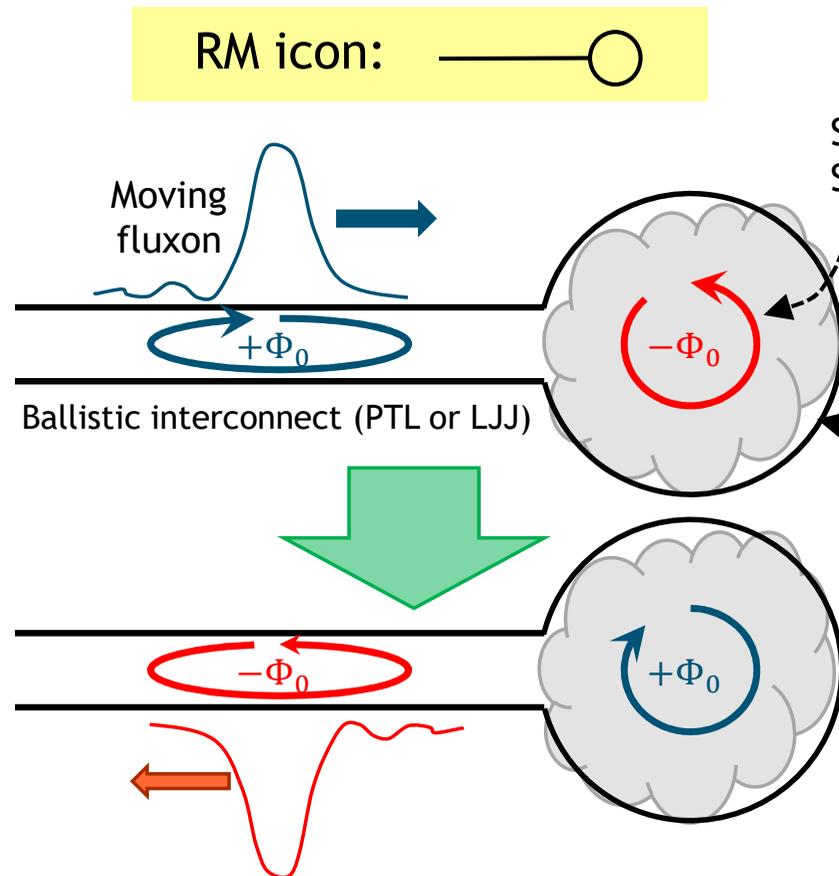


One of our early tasks: Characterize the simplest nontrivial BARC device functionalities, given a few simple design constraints applying to an SCE-based implementation, such as:

- (1) Bits encoded in fluxon polarity; (2) Bounded planar circuit conserving flux; (3) Physical symmetry.

Determined through theoretical hand-analysis that the simplest such function is the ***1-Bit, 1-Port Reversible Memory Cell (RM)***:

- Due to its simplicity, this was then the preferred target for our subsequent detailed circuit design efforts...



Stationary SFQ

Some planar, unbiased, reactive SCE circuit w. a continuous superconducting boundary

- Only contains L's, M's, C's, and *unshunted* JJs
- Junctions should mostly be *subcritical* (avoids  $R_N$ )
- Conserves total flux, approximately nondissipative

Desired circuit behavior (NOTE: conserves flux, respects T symmetry & logical reversibility):

- If polarities are opposite, they are swapped (shown)
- If polarities are identical, input fluxon reflects back out with no change in polarity (not shown)
- (*Deterministic*) *elastic 'scattering'* type interaction: Input fluxon kinetic energy is (nearly) preserved in output fluxon

RM Transition Table

Input Syndrome	→	Output Syndrome
+1(+1)	→	(+1)+1
+1(-1)	→	(+1)-1
-1(+1)	→	(-1)+1
-1(-1)	→	(-1)-1

# RM—First working (in simulation) implementation!



Erik DeBenedictis: “Try just strapping a JJ across that loop.”

- This actually works!

“Entrance” JJ sized to = about 5 LJJ unit cells ( $\sim 1/2$  pulse width)

- I first tried it twice as large, & the fluxons annihilated instead...
  - “If a  $15 \mu\text{A}$  JJ rotates by  $2\pi$ , maybe  $1/2$  that will rotate by  $4\pi$ ” 🤔

Loop inductor sized so  $\pm 1$  SFQ will fit in the loop (but not  $\pm 2$ )

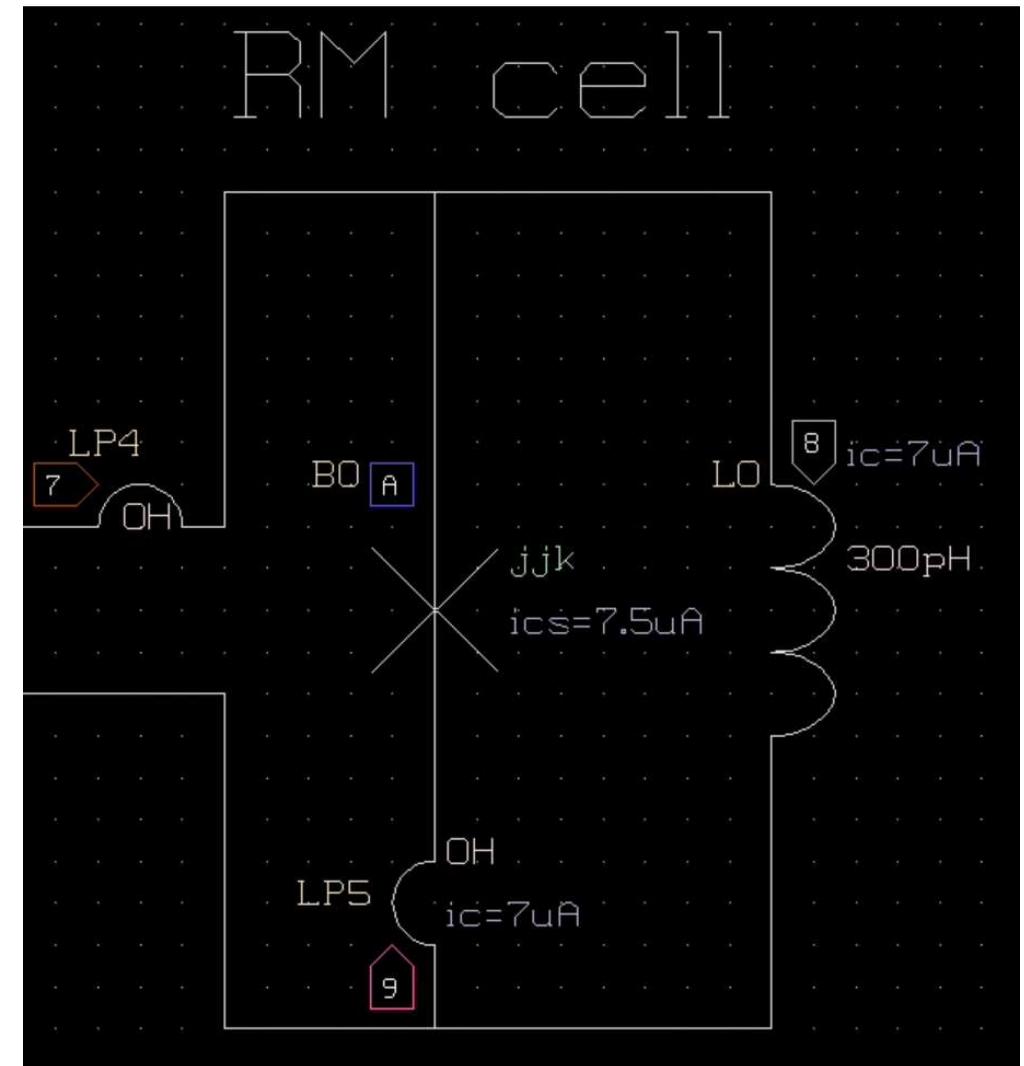
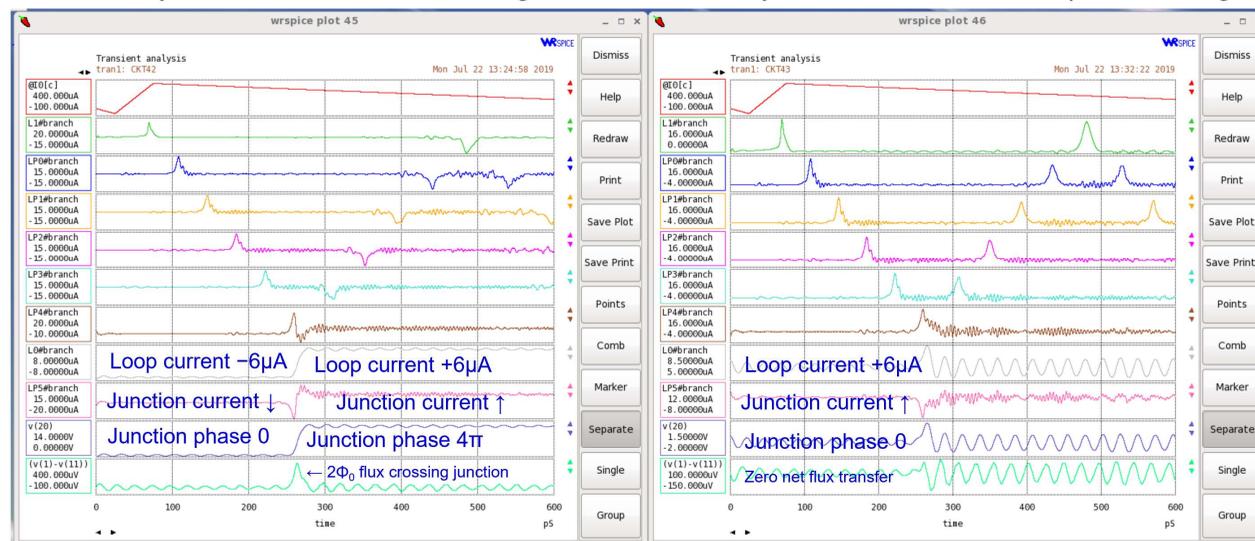
- JJ is sitting a bit below critical with  $\pm 1$

WRspice simulations with  $\pm 1$  fluxon initially in the loop

- Uses `ic` parameter, & `uic` option to `.tran` command
  - Produces initial ringing due to overly-constricted initial flux
  - Can damp w. small shunt  $G$

Polarity mismatch  $\rightarrow$  Exchange

Polarity match  $\rightarrow$  Reflect (=Exchange)

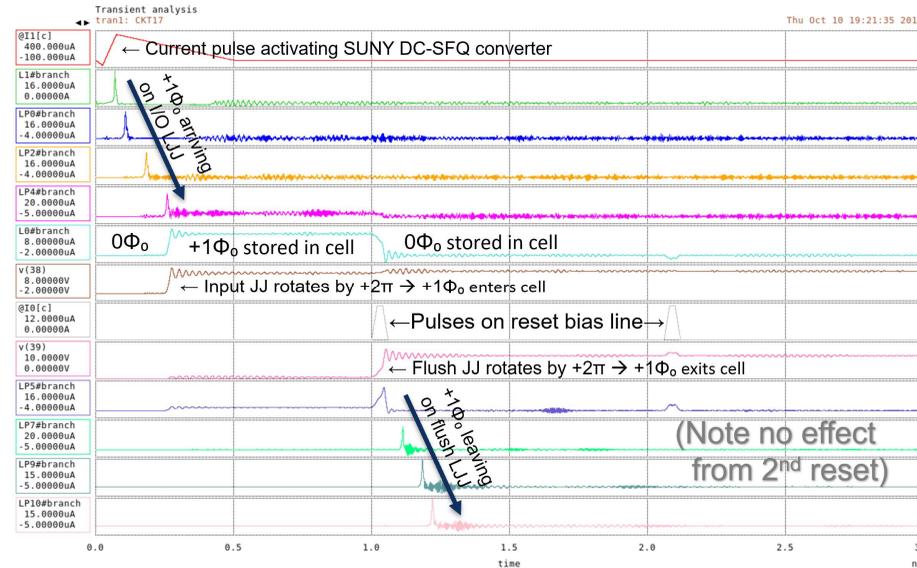
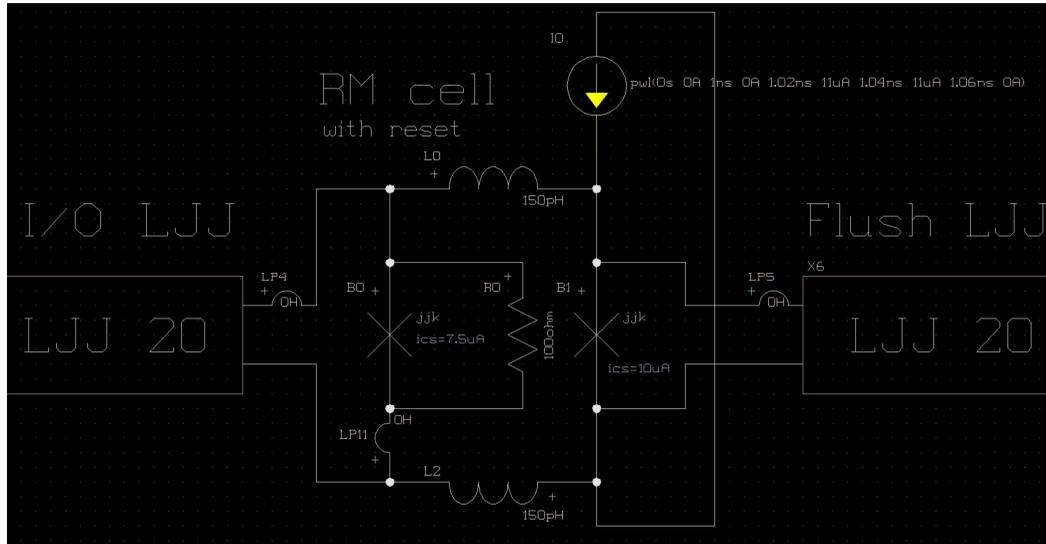


# Resettable version of RM cell—Designed & Fabricated!



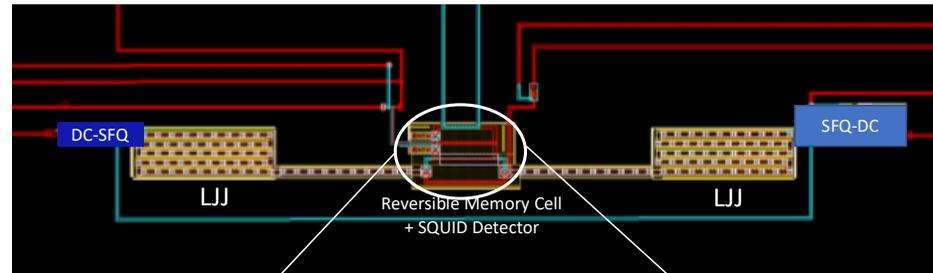
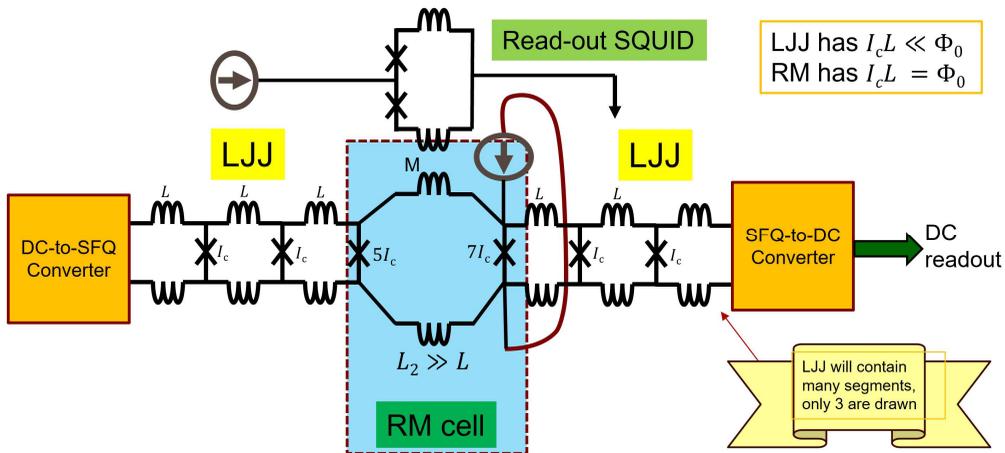
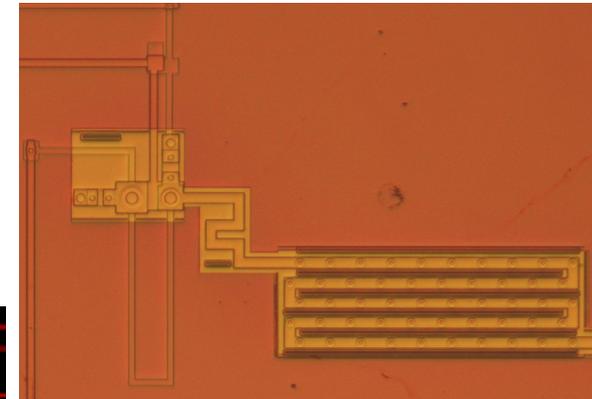
Apply current pulse of appropriate sign to flush the stored flux (the pulse here flushes out positive flux)

- To flush either polarity → Do both ( $\pm$ ) resets in succession

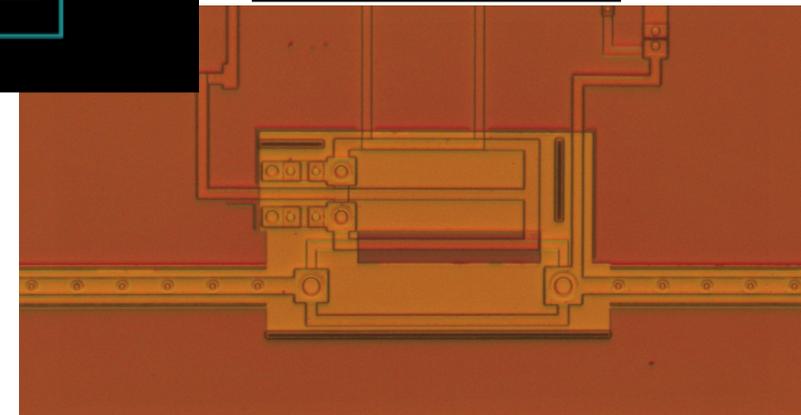
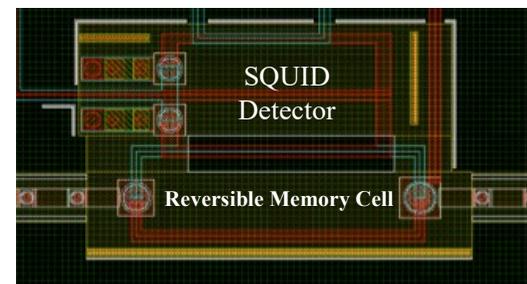


Fabrication at SeeQC with support from ACI

DC-SFQ & LJJ



RM Cell & SQUID



# barc tool for enumerating/classifying BARCS device functions



Custom Python program with 16 modules.

Tool is now complete; will be open-sourced.

Layer-cake view of software architecture:

- Modules only import modules from lower-numbered layers.

```
Symmetry group #38 has 6 functions:
Function #155.
Function #340.
Function #481.
Function #285.
Function #365.
Function #185.

Example: Function #155 = [1]*3(L,R):
1(L) -> (R)2
1(R) -> (L)3
2(L) -> (R)1
2(R) -> (R)3
3(L) -> (L)2
3(R) -> (L)1

Function #155 has the following symmetry properties:
It is D-dual to function #481
It is S-dual to function #481
It is E(1,2)-dual to function #340
It is E(1,3)-dual to function #185
It is E(2,3)-dual to function #481
It R(-1)-transforms to function #365
It R(1)-transforms to function #285
```

Layer	Module Names & Descriptions
4	<b>barc</b> (top-level program)
3	<b>deviceType</b> – Classification of devices with given dimensions.
2	<b>deviceFunction</b> – Device with a specific transition function. <b>stateSet</b> – Identifies a set of accessible device states.
1	<b>pulseAlphabet</b> – Sets of pulse types. <b>pulseType</b> – Identifies a specific type of pulse. <b>state</b> – Identifies an internal state of a device. <b>symmetryGroup</b> – Equivalence class of device functions. <b>transitionFunction</b> – Bijective map, input→output syndromes.
0	<b>characterClass</b> – Defines a type of signal characters. <b>deviceDimensions</b> – Defines size parameters of devices. <b>dictPermuter</b> – Used to enumerate transition functions. <b>signalCharacter</b> – Identifies I/O event type (pulse type & port). <b>symmetryTransform</b> – Invertibly transforms a device function. <b>syndrome</b> – An initial or final condition for a device transition. <b>utilities</b> – Defines some low-level utility functions.

← Example description of a symmetry-equivalence group as output by the **barc** tool.

# Symmetry Relations of Interest



The following symmetry relations on BARC functions are considered in this work:

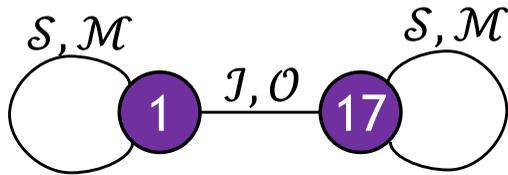
- ***Direction-reversal symmetry***  $\mathcal{D}$  – Symmetry under exchange of input & output syndromes (involution of transition func.)
- ***State-exchange symmetry***  $\mathcal{S}$  – Symmetry under an exchange of state labels (and fluxes, for flux-polarized states).
- ***Flux-negation symmetry***  $\mathcal{F}$  – Symmetry under negation of all (I/O flux & internal state) flux polarities.
- ***Moving-flux negation symmetry***  $\mathcal{M}$  – Symmetry under negation of all moving (I/O) flux polarities.
  - *Input flux negation symmetry*  $\mathcal{I}$  – Symmetry under negation of all input flux polarities.
  - *Output flux negation symmetry*  $\mathcal{O}$  – Symmetry under negation of all output flux polarities.
- ***Port-relabeling symmetries***  $\mathcal{R}_P$  – Symmetry under a particular permutation  $P$  of the port labels.
  - *Port exchange symmetry*  $\mathcal{E}(p_i, p_j)$  – Symmetry wrt an exchange of labels between a particular pair of ports.
  - *Rotational symmetry*  $\mathcal{R}_r$  – Relevant for  $n \geq 3$  ports. Symmetry under (planar) rotation of port labels.
  - *Reflection across port axis*  $\mathcal{R}_{\{p_i\}}$  – Symmetry under reflection of ports on either side of port  $p_i$ .
  - *Mirror symmetry*  $\mathcal{M}_2, \mathcal{M}_3$  – Symmetry under port exchange for a 2-port device, or any reflection for a rotationally symmetric 3-port device.
  - *Complete port symmetry*  $\mathcal{R}(n)$  – Symmetry under *all* possible relabelings of the ports.

# Equivalence Groups For the 24 One-Port, Two-State Elements:

$2 \cdot 1 \cdot 2 = 4$  I/O syndromes  $\rightarrow 4! = 24$  permutations (raw reversible transition functions).

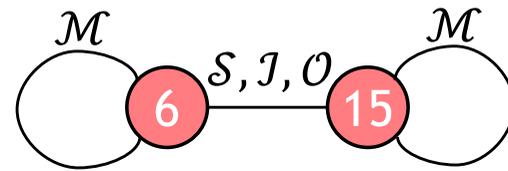


Stateful Reflector



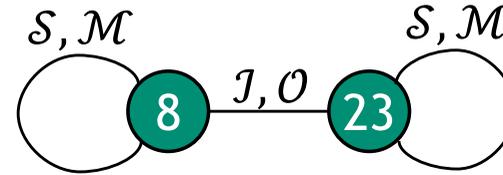
(State Unused—Not Atomic)

Configurable Inverter



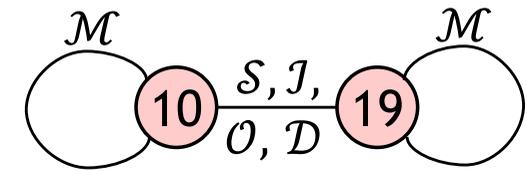
(Doesn't Change State)

Toggle



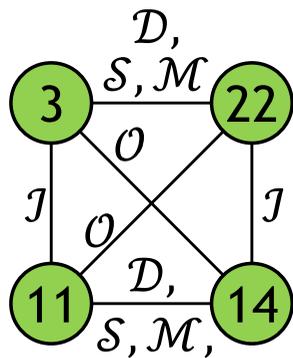
(Doesn't Use State)

Toggle & Conditional Invert

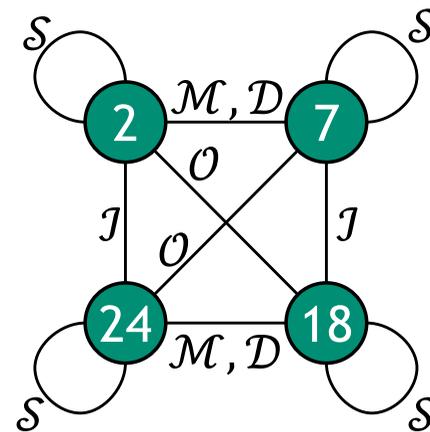


(Neither flux-negation symmetric nor flux-conserving)

Exchange (RM)

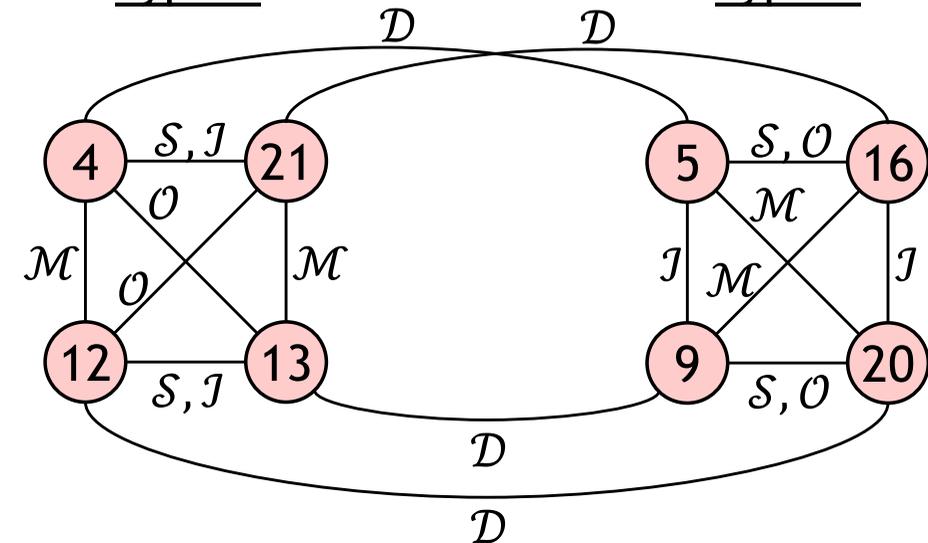


Conditional Toggle



(Doesn't Use State)

Type 4



(Neither flux-negation symmetric nor flux-conserving)

# Two-Port, Two-State, Flux-Polarized Elements

There are  $2^3 = 8$  I/O syndromes, thus  $8! = 40,320$  raw reversible transition functions.

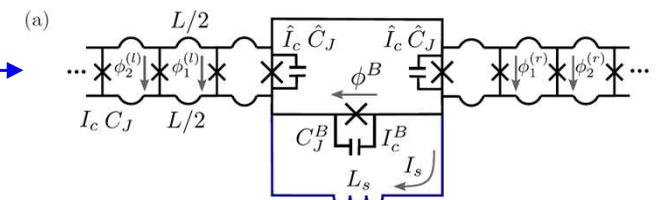
- But only 96 of them satisfy the flux conservation constraint.
- And only 10 of these are nontrivial primitives satisfying all constraints.

These 10 functions sort into 7 equivalence groups as follows:

Self-Symmetry Group Size	Equivalence Group Size	Number of Equiv. Groups	Total # of Raw Trans. Funcs.
4	1	4	4
2	2	3	6
<b>TOTALS:</b>		<b>7</b>	<b>10</b>

The corresponding functional behaviors can be described as:

1. Reversible Shift Register (RSR) – More on this one later.
2. Directed Reversible Shift Register (DRSR)
3. Filtering RM Cell (FRM)
4. Directed Filtering RM Cell (DFRM).
5. Polarized Flipping Diode (PFD). – Also has a flux-neutral equivalent.
6. Asymmetric Polarity Filter (APF).
7. Two-Port Reversible Memory Cell (RM2). – Implemented.



(Osborn & Wustmann '22)

# Illustrations of 2-port, 2-state, flux-polarized elements:

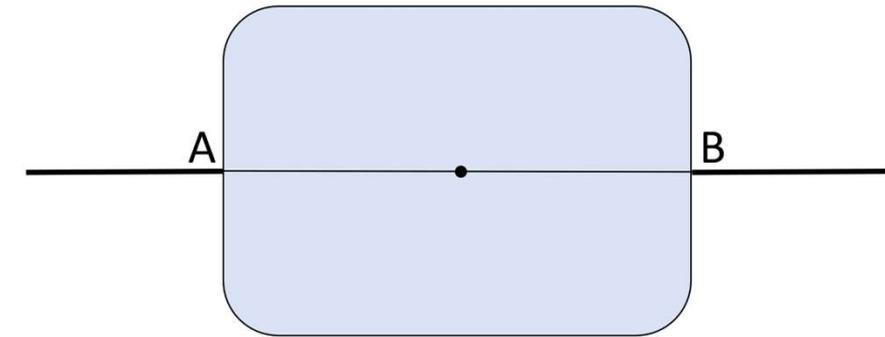


(Table Rows Shown for  $\uparrow$  Initial State Only)

## 1. Reversible Shift Register (RSR):

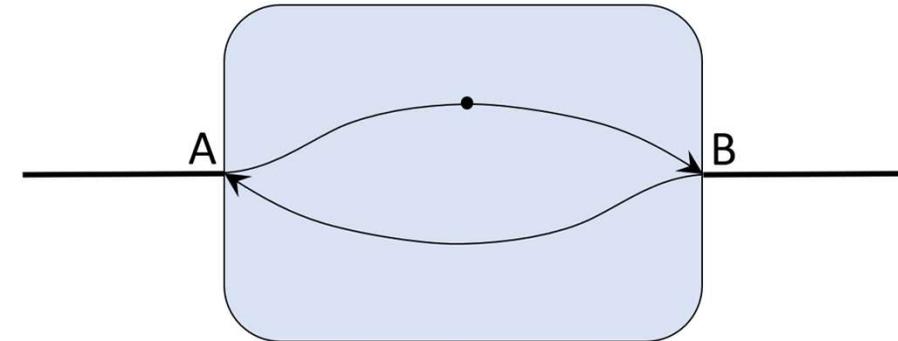
(Implemented by Osborn & Wustmann '22)

Input syndrome	Output syndrome
$\uparrow \rangle A(\uparrow)$	$(\uparrow) B \rangle \uparrow$
$\downarrow \rangle A(\uparrow)$	$(\downarrow) B \rangle \uparrow$
$\uparrow \rangle B(\uparrow)$	$(\uparrow) A \rangle \uparrow$
$\downarrow \rangle B(\uparrow)$	$(\downarrow) A \rangle \uparrow$



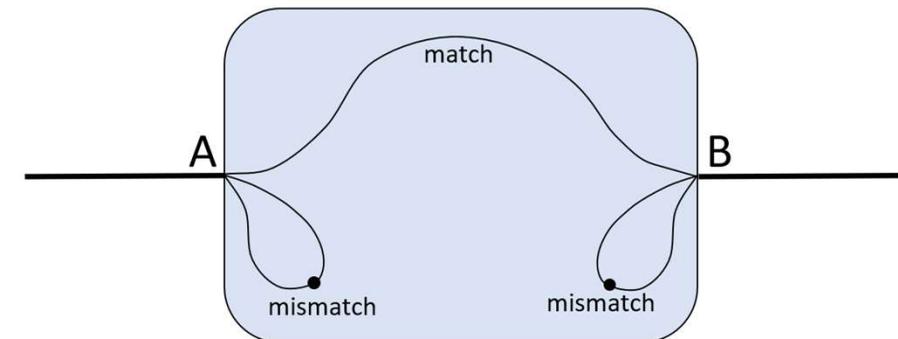
## 2. Directed Reversible Shift Register (DRSR):

Input syndrome	Output syndrome
$\uparrow \rangle A(\uparrow)$	$(\uparrow) B \rangle \uparrow$
$\downarrow \rangle A(\uparrow)$	$(\downarrow) B \rangle \uparrow$
$\uparrow \rangle B(\uparrow)$	$(\uparrow) A \rangle \uparrow$
$\downarrow \rangle B(\uparrow)$	$(\uparrow) A \rangle \downarrow$



## 3. Filtering RM Cell (FRM):

Input syndrome	Output syndrome
$\uparrow \rangle A(\uparrow)$	$(\uparrow) B \rangle \uparrow$
$\downarrow \rangle A(\uparrow)$	$(\downarrow) A \rangle \uparrow$
$\uparrow \rangle B(\uparrow)$	$(\uparrow) A \rangle \uparrow$
$\downarrow \rangle B(\uparrow)$	$(\downarrow) B \rangle \uparrow$



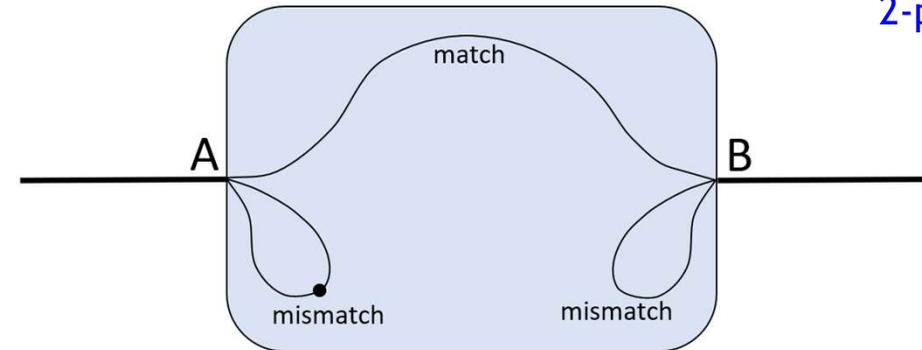
# Illustrations of 2-port, 2-state, flux-polarized elements, cont.:



(Not shown:  
2-port RM cell)

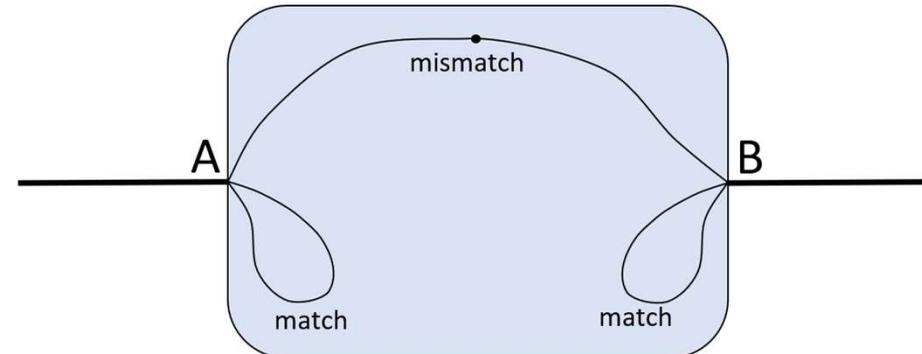
## 4. Directed Filtering RM Cell (DFRM):

Input syndrome	Output syndrome
$\uparrow\rangle A(\uparrow)$	$(\uparrow)B\rangle \uparrow$
$\downarrow\rangle A(\uparrow)$	$(\downarrow)A\rangle \uparrow$
$\uparrow\rangle B(\uparrow)$	$(\uparrow)A\rangle \uparrow$
$\downarrow\rangle B(\uparrow)$	$(\uparrow)B\rangle \downarrow$



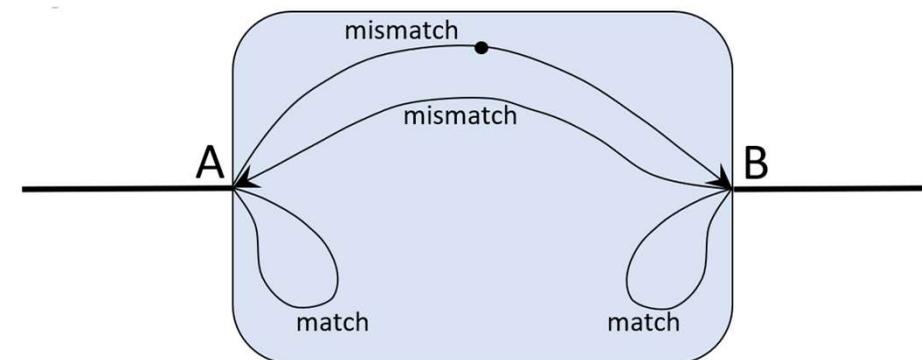
## 5. Polarized Flipping Diode (PFD):

Input syndrome	Output syndrome
$\uparrow\rangle A(\uparrow)$	$(\uparrow)A\rangle \uparrow$
$\downarrow\rangle A(\uparrow)$	$(\downarrow)B\rangle \uparrow$
$\uparrow\rangle B(\uparrow)$	$(\uparrow)B\rangle \uparrow$
$\downarrow\rangle B(\uparrow)$	$(\downarrow)A\rangle \uparrow$



## 5. Asymmetric Polarity Filter (APF):

Input syndrome	Output syndrome
$\uparrow\rangle A(\uparrow)$	$(\uparrow)A\rangle \uparrow$
$\downarrow\rangle A(\uparrow)$	$(\downarrow)B\rangle \uparrow$
$\uparrow\rangle B(\uparrow)$	$(\uparrow)B\rangle \uparrow$
$\downarrow\rangle B(\uparrow)$	$(\uparrow)A\rangle \downarrow$



# Two-Port, Two-State, Flux-Neutral Elements

There are  $(2^2)! = 24$  raw flux-symmetric transition functions.

- 14 of these are nontrivial, atomic functional primitives.

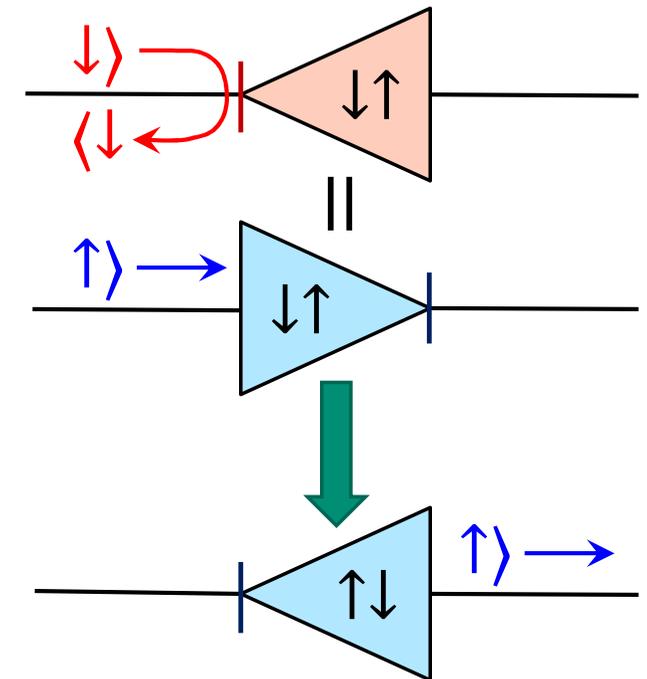
These sort into 4 equivalence groups as follows:

Self-Symmetry Group Size	Equivalence Group Size	Number of Equiv. Groups	Total # of Raw Trans. Funcs.
4	2	3	6
1	8	1	8
<b>TOTALS:</b>		<b>4</b>	<b>14</b>

There are 5 distinct functional behaviors (described in forwards time direction):

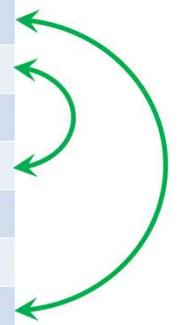
- Alternating Barrier (AB)**, 2 representations – See next slide.
- Polarized Flipping Diode (PFD)**, 2 reps..
- Variant Polarized Flipping Diode (VPFD)**, 2 reps..
- Asymmetric Polarized Flipping Diode (APFD)**, 4 reps.,  
(and this one is  $\mathcal{D}$ -dual to:)
- Selectable Barrier (SD)**, 4 reps.

Ex: Polarized Flipping Diode (PFD)



**Polarity-Dependent Flipping Diode (PFD)**

I/O Syndrome		
Port	State	Fluxon
Left	$\uparrow\downarrow$	$\uparrow$
Left	$\uparrow\downarrow$	$\downarrow$
Left	$\downarrow\uparrow$	$\uparrow$
Left	$\downarrow\uparrow$	$\downarrow$
Right	$\uparrow\downarrow$	$\uparrow$
Right	$\uparrow\downarrow$	$\downarrow$
Right	$\downarrow\uparrow$	$\uparrow$
Right	$\downarrow\uparrow$	$\downarrow$



# Ex. 2-port, 2-state neutral element: Alternating Barrier (AB)

Flux-conserving, flux-negation symmetric element.

- Also has mirror ( $\mathcal{M}_2$ ) symmetry.
- Has two  $\mathcal{D}, \mathcal{S}$  dual representations.

Flux-neutral internal states  $\rightarrow$  Doesn't change fluxon polarity.

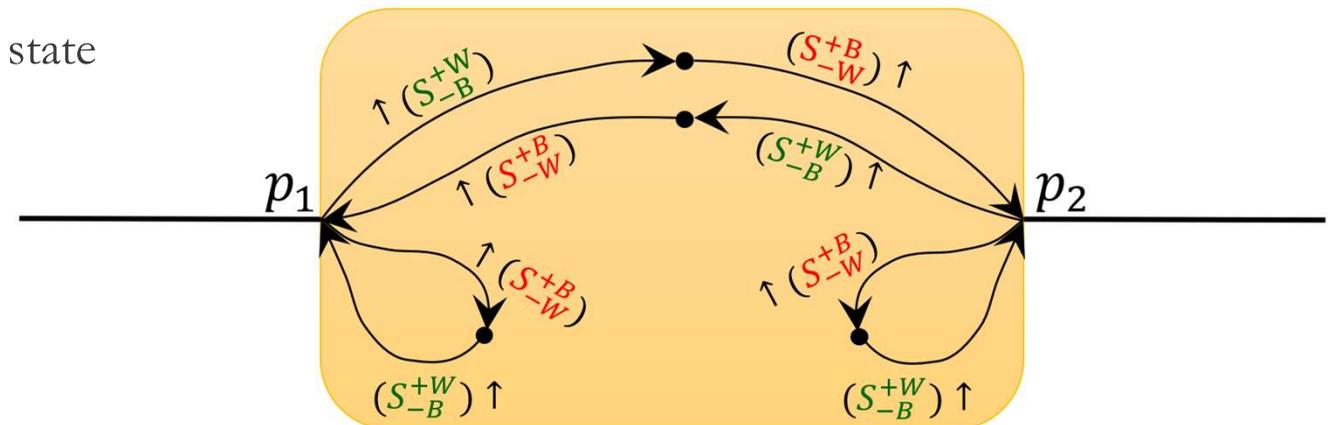
State descriptions:

- $S_{-B}^{+W}$ : *Positive-wire, negative-barrier.*
  - Transmits positive ( $\uparrow$ ) fluxons, reflects negative ( $\downarrow$ ) fluxons.
- $S_{-W}^{+B}$ : *Positive-Barrier, negative-wire.*
  - Reflects positive ( $\uparrow$ ) fluxons, transmits negative ( $\downarrow$ ) fluxons.

Transition function description:

- Fluxons arriving at either port are routed as per the state descriptions above.
- State toggles with every interaction.

Input Syndrome	Output Syndrome
$\uparrow\rangle p_1 (S_{-B}^{+W})$	$(S_{-W}^{+B}) p_2 \rangle \uparrow$
$\downarrow\rangle p_1 (S_{-B}^{+W})$	$(S_{-W}^{+B}) p_1 \rangle \downarrow$
$\uparrow\rangle p_2 (S_{-B}^{+W})$	$(S_{-W}^{+B}) p_1 \rangle \uparrow$
$\downarrow\rangle p_2 (S_{-B}^{+W})$	$(S_{-W}^{+B}) p_2 \rangle \downarrow$
$\uparrow\rangle p_1 (S_{-W}^{+B})$	$(S_{-B}^{+W}) p_1 \rangle \uparrow$
$\downarrow\rangle p_1 (S_{-W}^{+B})$	$(S_{-B}^{+W}) p_2 \rangle \downarrow$
$\uparrow\rangle p_2 (S_{-W}^{+B})$	$(S_{-B}^{+W}) p_2 \rangle \uparrow$
$\downarrow\rangle p_2 (S_{-W}^{+B})$	$(S_{-B}^{+W}) p_1 \rangle \downarrow$



# Summary of Results for Three-Port, Two-State Elements:



(Still assuming flux conservation & flux negation symmetry)

Devices with flux-polarized states:

- $2 \cdot 3 \cdot 2 = 12$  I/O syndromes
- $12! = 497,001,600$  raw reversible funcs.
- 25,920 of these are flux-conserving.
- 288 of those are flux-negation symmetric.
- 245 of those are atomic (primitives).
- 219 of those use the state non-trivially.
- Sort into 39 equiv. groups as follows →

Devices with flux-neutral states:

- $1 \cdot 3 \cdot 2 = 6$  I/O syndromes (for ↑ inputs)
- $6! = 720$  permutations.
- 653 of them are atomic primitives.
- 600 of those use the state non-trivially.
- Sort into 45 equiv. groups as follows:

## Summary of (3,2) flux-polarized behaviors

<b>Equivalence Class Size:</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>6</b>	<b>12</b>	<b>Tot.</b>
<b>Self-Symmetry Group Size:</b>	<b>12</b>	<b>6</b>	<b>4</b>	<b>2</b>	<b>1</b>	
No. of Equivalence Classes:	1	4	6	24	4	<b>39</b>
Total number of Functions:	1	8	18	144	48	<b>219</b>

## Summary of (3,2) flux-neutral behaviors

<b>Equivalence Class Size:</b>	<b>2</b>	<b>4</b>	<b>6</b>	<b>12</b>	<b>24</b>	<b>Tot.</b>
<b>Self-Symmetry Group Size:</b>	<b>12</b>	<b>6</b>	<b>4</b>	<b>2</b>	<b>1</b>	
No. of Equivalence Classes:	1	1	9	23	11	<b>45</b>
Total number of Functions:	2	4	54	276	264	<b>600</b>

# Illustrations of some 3-port, 2-state flux-neutral elements

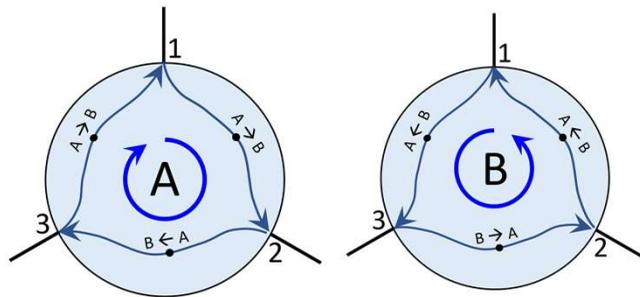


Recall there are 45 different non-trivial, atomic functional behaviors (counting  $\mathcal{D}$ -duals as equivalent).

Of these, only a few exemplar behaviors are illustrated here.

Still seeking implementations of any of these....

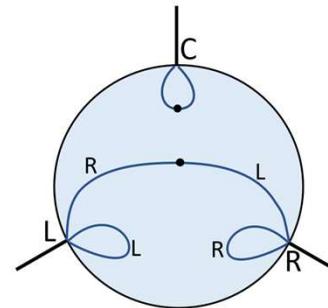
## Polarized Neutral Toggle Rotary (PNTR)



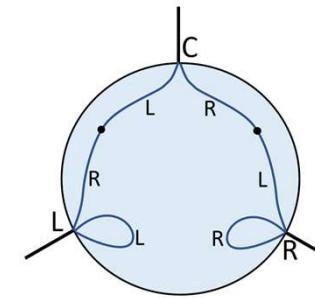
Behavior in Positive-Clockwise State (A)

Behavior in Positive-Counterclockwise State (B)

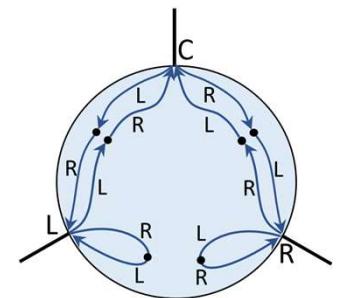
## Polarized Controlled Flipping Diode (PCFD)



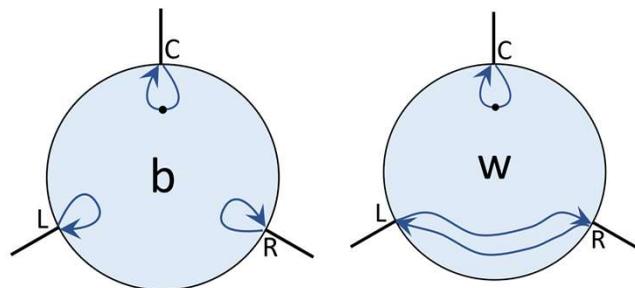
## Polarized Throw Switch, Type A (PTSA)



## Polarized Throw Switch, Type B (PTSB)



## Polarized Toggle Controlled Barrier (PTCB)

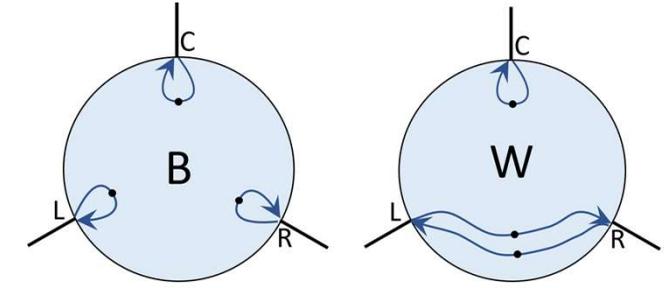


Behavior in Positive Barrier State (b)

Behavior in Positive Wire State (w)

*[NOTE: All behaviors shown here are for (+) fluxons only; (-) fluxons interact oppositely with states]*

## Polarized Knock-twice Toggle Controlled Barrier (PKTCB)



Behavior in Positive Barrier State (B)

Behavior in Positive Conducting State (W)

## Some Next Steps for the BARCS effort



1. Document classification results more fully (in progress).
2. Finish developing **SCIT** (Superconducting Circuit Innovation Tool) tool to facilitate discovery of circuit-level implementations of BARCS functions.
  - Including training an AI/ML model to quickly solve the inverse (circuit design) problem.
3. Better understand role of physical symmetries in the circuit design of BARCS elements.
  - What, if any, functions are ruled out by the symmetries?
  - Must we consider including additional SCE device types to break the symmetries?
4. Identify a computation-universal set of primitive elements that we also know how to implement!
  - Or, show that this is impossible using the present set of devices.
5. Additional work on fabrication & empirical validation of BARCS circuit designs.
6. Gain a better understanding of the limits of the energy efficiency of this approach.

Clearly, much work along these lines remains to be done!

- We would be very happy to recruit new collaborators

## Conclusion



The long-neglected *ballistic* mode of reversible computing has recently attracted renewed interest.

- Classic problems with synchronization & chaotic instability in ballistic computing schemes appear to be resolvable via the asynchronous approach.
- The new method seems to hold some promise for possibly achieving improved energy-delay products and/or more compact circuit designs vs. adiabatic approaches.

Also, note that ballistic approaches are not viable at all in CMOS!

- CMOS has nothing like a ballistic flux soliton, & has no nonlinear reactive elements like JJs...
- Thus, we are leveraging unique advantages of superconducting electronics in this approach.

In this paper & talk, we reported our progress on enumerating & classifying the possible BARCS functions...

- Given constraints of full logical reversibility, flux conservation, & flux negation symmetry.

Multiple US-based research groups in superconductor physics & engineering are now making early progress along this line of work...

- We invite additional domestic & international colleagues to join us in investigating this interesting line of research!